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Research Article

Application of the Homotopy Perturbation Method for Analysing the Spread of the Nipah Virus

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I N F O

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A B S T R A C T

Introduction: In this article, we have analyzed the mathematical model of the non-linear differential equation of the Nipah virus (NiV). It was developed by Jakia Sultana et al. to investigate the optimal control of the virus. The transmission dynamics of the Nipah virus were analyzed both qualitatively and numerically.

Method: We examined the values of the variation in parameters to investigate the impact of the controls on the spread of the disease. The birth rate, mortality rate, contact rate (β), and NiV-induced death rates (α) are discussed.

Results: Through the numerical and graphical results, we have discussed optimal control, which helps to significantly reduce the impact of the disease. A significant agreement is produced when approximate analytical results are compared to numerical simulations.

Conclusion: The study highlights the effectiveness of optimal control strategies in mitigating the spread of the Nipah virus.

Keywords: Nipah Virus (NiV), Optimal Control, Human Population, Infectious Disease, Homotopy Perturbation Method, Numerical Simulation

Introduction

Infectious illness analysis and control now frequently use mathematical modelling as a key tool. It can also be used to assess how various approaches to preventing the spread of infectious illnesses within a population are faring. "The theory and practice of disease management and control have been increasingly influenced in recent years by epidemiological modelling of infectious disease transmission".[1] Herein, the goal is to reduce the number of sick people while increasing the total number of people who recover. A developing virus in the southeast and south-Asian regions, the Nipah virus is a member of the genus Henipavirus, a new class of viruses in the Paramyxoviridae family.[2] The frequent outbreaks and high fatality rate of this newly developing infectious disease have made it one of the most concerning risks to public health.[3] "An application of epidemiology is the management of health issues. Epidemiology is the study of the distribution and determinants of health-related states or events in particular populations". The key distinction between clinical medicine and epidemiology is that clinical

medicine works with individuals, whereas epidemiology is concerned with populations or groups of persons (patients). "Epidemiology, therefore, describes health and disease in terms of frequency and distributions of determinants and circumstances in a population or a particular group of a population". Although there have been only a few outbreaks of the Nipah virus, it is a public health concern since it affects a variety of animals, causes serious illness in humans, and even results in death in some cases.[4] As the effectiveness of antiviral medications is unsatisfactory, the majority of treatments are symptomatic and supportive. In light of the extremely high case fatality rate, effective and stringent control and preventive measures are needed. This study applies an optimal control total dynamic model of infections caused by the Nipah virus (NiV). In this study, we use homotopy perturbation techniques to analyse the mathematical modelling of the spread of Nipah Virus (NiV) using optimal control technique.[5] Unlike other published conclusions in the literature, this one has not yet been mathematically investigated. Additionally, a comparison between the mathematical and numerical results has not yet been made, since there is no method to discover the solution to the system of non-linear differential equations used in the constructed model in the referred paper [5], we have attempted to locate the solution using semianalytical methods such as the homotopy perturbation method. According to the model analysis, Jakia Sultana et al. discovered the prerequisites for ideal control.[5] Our findings provide the finest understanding of Nipah virus propagation when applying control settings $\alpha, \beta, v, \mu\alpha, \beta, v, \mu$.

Mathematical Formulation of the Nipah Virus

Sultana and Podder developed the "Mathematical Analysis of Nipah Virus Infections Using Optimal Control Theory".[5] N(t) denotes the total human population at time t. We subdivide N(t) into three subpopulations, namely, susceptible individuals S(t), infected individuals I(t), and recovered individuals R(t). The mathematical modelling formulation of the Nipah virus[5] is described as follows:

$$S'(t) = vN(t) - \beta S(t)I(t) - (\mu + u_1)S(t) \quad (1)$$

$$I'(t) = \beta S(t)I(t) - (\gamma + \mu + \alpha)I(t) - u_2\mathrm{I}(t) \quad (2)$$

$$R'(t) = \gamma I(t) - \mu R(t) + u_1 S(t) + u_2\mathrm{I}(t) \quad (3)$$

$$N'(t) = vN(t) - \alpha I(t) - \mu N(t) \quad (4)$$

The initial conditions are:

$$S(0) = S_0 \geq 0, I(0) = I_0 \geq 0, R(0) = R_0 \geq 0, N(0) = N_0 \quad (5)$$

The following table presents the parametric values that are used in this model and are referred from the Mathematical analysis of Nipah virus infections using optimal control theory.[5]

Table 1.Description and Parameter Values of the NiV Model[5]

Symbol	Description	Values
v	Birth rate[5]	0.030
μ	Mortality rate[5]	0.002
α	Disease-induced death rate[5]	0.010
β	Contact rate[5]	0.750
γ	Recovery rate[5]	0.005
u_1	Weight parameter[5]	0.200
u_2	Weight parameter[5]	0.300

Approximate Analytical Expression of Nipah Virus Using the Homotopy Perturbation Method

Homotopy Perturbation Method (HPM) is used to solve non-linear Equations 1–5.

$$(1-V)\left(\frac{dS}{dt} + (\mu + u_1)S(t)\right) + V\left(\frac{dS}{dt} - vN(t) + \beta S(t)I(t) + (\mu + u_1)S(t)\right) = 0 \quad (6)$$

$$(1-\mathrm{V})\left(\frac{\mathrm{dI}}{\mathrm{dt}} + (\gamma + \mu + \alpha + u_2)\mathrm{I}(t)\right) + \mathrm{V}\left(\frac{\mathrm{dI}}{\mathrm{dt}} - \beta S(t)I(t) + (\gamma + \mu + \alpha + u_2)\mathrm{I}(t)\right) = 0 \quad (7)$$

$$(1-V)\left(\frac{\mathrm{dR}}{\mathrm{dt}} + \mu R(t)\right) + V\left(\frac{\mathrm{dR}}{\mathrm{dt}} - \gamma I(t) + \mu R(t) - u_1 S(t) - u_2\mathrm{I}(t)\right) = 0 \quad (8)$$

$$(1-V)\left(\frac{\mathrm{dN}}{\mathrm{dt}} - vN(t) + \mu N(t)\right) + V\left(\frac{\mathrm{dB_c}}{\mathrm{dt}} - vN(t) + \alpha I(t) + \mu N(t)\right) = 0 \quad (9)$$

The solution of Equations 12–16 is expressed in power series:

$$S = S_0 + VS_1 + V^2S_2 + V^3S_3 + \cdots \quad (10)$$

$$I = I_0 + VI_1 + V^2I_2 + V^3I_3 + \cdots \quad (11)$$

$$R = R_0 + VR_1 + V^2R_2 + V^3R_3 + \cdots \quad (12)$$

$$N = N + VN + V^2N + V^3N + \cdots \quad (13)$$

Substituting Equations 10–13 into Equations 6–9 and arranging the coefficients of the powers of V produces the following systems of differential equations:

$$\begin{aligned} V^0&: \frac{dS_0}{dt} + (\mu + u_1)S_0 \\ V^1&: \frac{dS_1}{dt} + (\mu + u_1)S_1 - vN_0 + \beta S_0 I_0 \\ V^2&: \frac{dS_2}{dt} + (\mu + u_1)S_2 - vN_1 + \beta S_0 I_1 + \beta S_1 I_0 \end{aligned} \quad (14)$$

$$\begin{aligned} V^0&: \frac{dI_0}{\mathrm{dt}} + (\gamma + \mu + \alpha + u_2)\mathrm{I}_0 \\ V^1&: \frac{dI_1}{\mathrm{dt}} + (\gamma + \mu + \alpha + u_2)\mathrm{I}_1 - \beta S_0 I_0 \\ V^2&: \frac{dI_2}{\mathrm{dt}} + (\gamma + \mu + \alpha + u_2)\mathrm{I}_2 - \beta S_0 I_1 - \beta S_1 I_0 \end{aligned} \quad (15)$$

$$\begin{aligned} V^0&: \frac{d\mathrm{R}_0}{\mathrm{dt}} + \mu\mathrm{R}_0 \\ V^1&: \frac{d\mathrm{R}_1}{\mathrm{dt}} + \mu\mathrm{R}_1 - \gamma\mathrm{I}_0 - u_1 s_0 - u_2\mathrm{I}_0 \\ V^2&: \frac{d\mathrm{R}_2}{\mathrm{dt}} + \mu\mathrm{R}_2 - \gamma\mathrm{I}_1 - u_1 s_1 - u_2\mathrm{I}_1 \end{aligned} \quad (16)$$

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$$V^0: \frac{dN_0}{dt} - \upsilon N_0 + \mu N_0$$
$$V^1: \frac{dN_1}{dt} - \upsilon N_1 + \mu N_1 + \alpha I_0 \quad (17)$$
$$V^2: \frac{dN_2}{dt} - \upsilon N_2 + \mu N_2 + \alpha I_1$$

Analytical Expression of Nipah Virus (NiV)

In this section, a new approach homotopy perturbation method is used to solve the analytical solution of Equations 1–5, and the expanded derivative of the HPM is discussed in Appendices A and B.

$$S(t) = 0.0581e^{-0.202t} + 0.1298e^{0.028t} - 0.0022e^{-0.317t} + 0.12017e^{-0.836t} + 0.3186e^{-0.289t} - 0.6116e^{-0.721t} + 0.3602e^{-0.519t} + 0.1926e^{-0.923t} + 0.0011e^{-1.153t} + 0.0227e^{-0.606t} + 0.0701e^{-0.491t} - 0.0002e^{-0.634t} + 0.2897e^{-0.261t} + 0.0478e^{-1.24t} + 0.0557e^{-0.693t} + 0.0102e^{-0.808t} - 0.0012e^{-1.355t} - 0.1612e^{-1.038000000*t} \quad (18)$$

$$I(t) = 0.3052e^{-0.261t} - 0.0537e^{-1.24t} - 0.1164e^{-0.491t} + 0.0014e^{-1.355t} - 0.0013e^{-1.153t} - 0.1468e^{-0.836t} + 0.9956e^{-0.289t} - 0.9972e^{-0.317t} + 0.7854e^{-0.721t} - 0.0317e^{-0.606t} - 0.5634e^{-0.519t} + 0.0002e^{-0.634t} + 0.1868e^{-1.038t} - 0.2291e^{-0.923t} - 0.0727e^{-0.693t} - 0.0126e^{-0.808t} \quad (19)$$

$$R(t) = -0.0037e^{-0.002t} - 0.2102e^{-0.317t} - 0.6631e^{-0.202t} + 0.0574e^{-0.519t} + 0.8696e^{0.028t} \quad (20)$$

$$N(t) = 0.9968e^{0.028t} + 0.0063e^{-0.317t} - 0.00306e^{-0.519t} \quad (21)$$

Results and Discussion

The numerical solution and the resultant analytical results were compared, which is represented graphically in Figures 1–5.

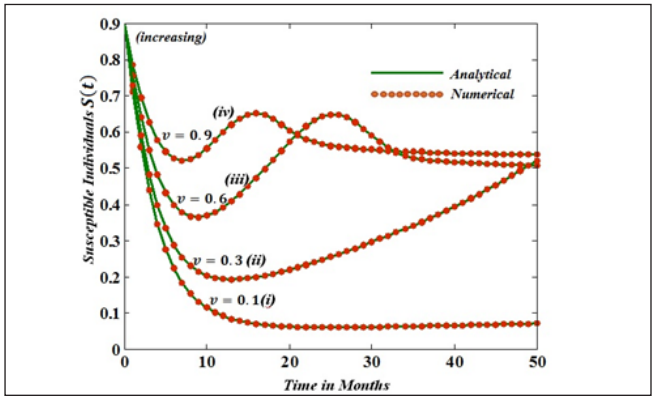

Figure 1.Variation in the Susceptible Individual Birth Rate

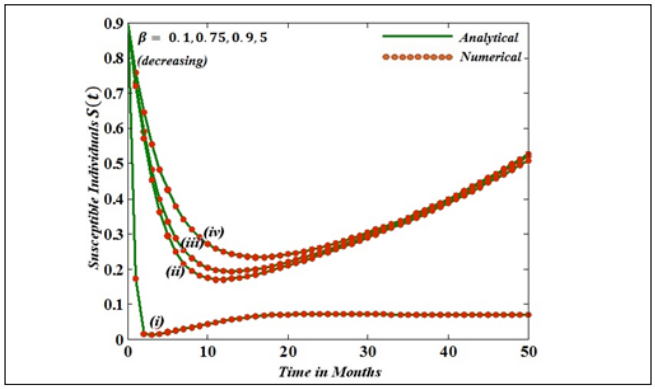

Figure 2.Variation in Susceptible Individual Contact Rate

In Figure 1, the susceptible individual's birth rate is increased due to changing the parameter values in the higher range as $\upsilon = 0.1, 0.3, 0.6$,and 0.9. Therefore, an increase in the birth rate creates an increase or decrease in the transmission rate. In Figure 2, the transmission rate or contact rate is reduced if we use proper prevention and control. While varying the values of $\beta = (i)0.1,\ (ii)0.75,\ (iii)0.9\ and\ (iv)5$, the transmission rate is reduced. Therefore, we should maintain a hygienic lifestyle to prevent the virus. If we follow this procedure, the infected human individual's rate will be decreased, as shown in Figure 2.

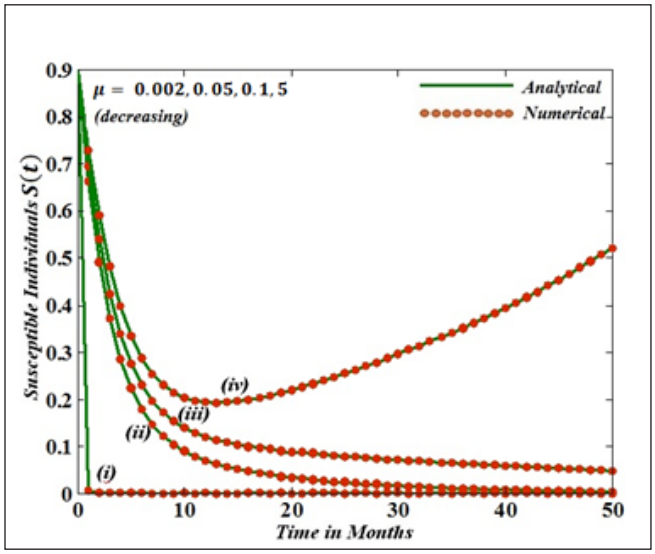

Figure 3.Variation in Susceptible Individual Mortality Rate

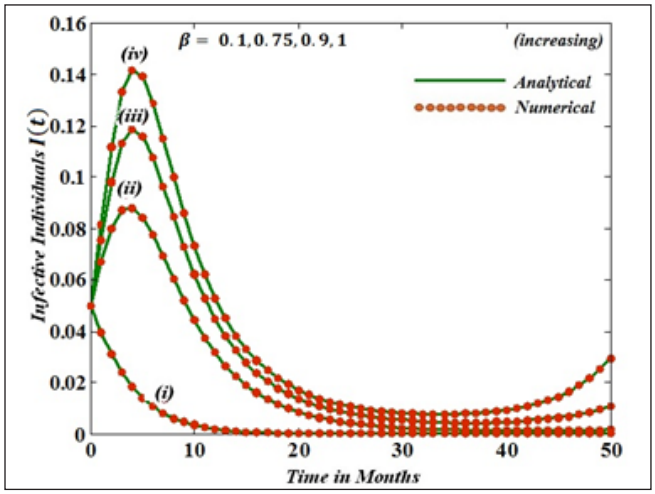

Figure 4.Variation in Susceptible Individuals Infected Individuals

In Figure 3, the mortality rate of susceptible individuals is decreased, while increasing the value of $\mu = (i)0.002(ii)0.05(iii)0.1(iv)5$ and in Figure 4, the rate of infected individuals increases while the value of the transmission rate is increased as . $\beta = (i)0.1\ (ii)0.75\ (iii)0.9\ and(iv)\ 1$.

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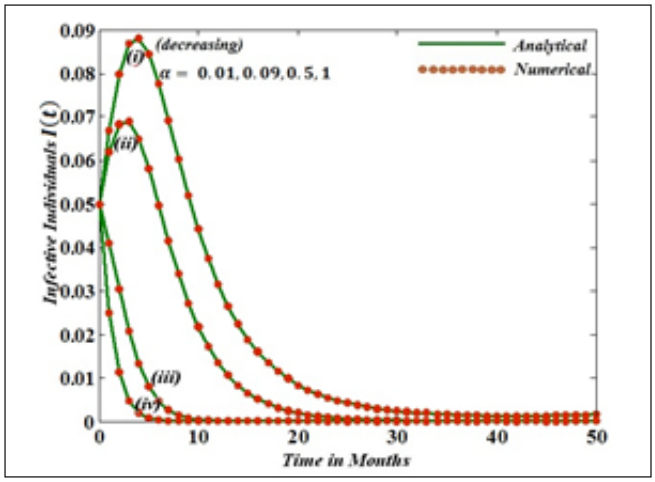

Figure 5.Variation in the Infected Population-Induced Death Rate

In the infected population, the Nipah virus-induced death rate is decreased if we increase the values of α to (i) 0.01(ii) 0.09 (iii) 0.5, and (iv) 1. With the practice of hand washing regularly with soap and water, sanitary facilities, and sufficient medical care, avoid going to places where bats are known to roost and avoid consuming raw date palm sap, unwashed fruit, or fruit that has fallen to the ground if you think it may have been contaminated by bats. Standard infection control procedures and effective barrier nursing techniques are crucial for preventing hospital-acquired illnesses in situations when a patient has a confirmed or suspected NiV infection since NiV can be transferred from person to person.

Conclusion

The governing equation of Nipah virus infection consists of a system of non-linear differential equations; it contains susceptible S(t), infectious I(t), and recovery R(t) compartment models concerning time, whose initial values are greater than zero, and N(t) is the sum of S(t), I(t) and R(t).

The non-linear differential equation is solved using the analytic approach of the homotopy perturbation method (HPM). In this paper, the Nipah virus model is solved analytically by using the HPM method, and the resulting results accord well with the numerical findings. The MATLAB ode45 function generates numerical results that are consistent with the analytical solution by the homotopy perturbation method. We have represented parameter values that flow graphically in MATLAB. The variation in the parameters in a graph indicates the possibilities of the transmission rate, natural death rate, and induced death rate going high or low, while the range of parameter values increases.

To reduce the number of infected people and control the dynamics of the disease, the effect of control parameters is quite notable. In cases of high incidence, the controls must be effective for a longer period. The best control is far more effective at reducing the number of affected people. Educating those in populations at increased risk of contracting NiV about its symptoms, indications, and risk of goods contaminated by fruit bats or contact with fruit bats Animals that could come into touch with fruit bats, such as pigs, should be avoided. Work in healthcare or as a caregiver for NiV-infected individuals.

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Conflict of Interest: None

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Appendix A

The Homotopy Perturbation Method's Fundamental Principle

Consider the following non-linear functional equation to demonstrate the essential elements of this technique:

$$X(u) - Y(v) = 0, s \in \Omega \quad \text{....(A1)}$$

$$\cup\left(u, \frac{\partial u}{\partial t}\right) = 0, v \in \Gamma \quad \text{....(A2)}$$

X indicates the functional operator, Y is a boundary operator, Y(v) for a known analytical equation and Ω the limit of the domain Γ . M and R, with M denoting linear and R denoting non-linear.

$$M(u) + R(u) - Y(v) = 0 \quad \text{.... (A3)}$$

We create homotopy equations using the homotopy procedure,

$$K(u,V) = (1-V)[M(u) - M(u_0)] + V(X(u) - Y(v)] = 0, V \in [0,1], v \in \Omega \quad \text{....(A4)}$$

or

$$K(u,V) = M(u) - M(u_0) + VM(u_0) + V[R(u) - Y(v)] = 0 \quad \text{.... (A5)}$$

is a parameter, For the solution of the equation, is an initial approximation. (A2), which meets the border requirements. Clearly, we may deduce the following from Equations A4 and A5:

$$K(u,0) = M(u) - M(u_0) = 0 \quad \text{.... (A6)}$$

$$K(u,1) = X(u) - Y(v) = 0 \quad \text{.... (A7)}$$

Altering V's value from zero to unity corresponds to changing value from to . This is known as homotopy in topology. The embedding parameter can be used as a tiny parameter, and the solution of Equations A4 and A5 can be assumed to be a power series in V:

$$U = u_0 + Vu_1 + V^2u_2 + \ldots \quad \text{.... (A8)}$$

Setting V = 1, an approximation to the solution of Equation A8

$$u = \lim_{Q \to 1} U = u_0 + u_1 + u_2 + \cdots \quad \text{.... (A9)}$$

The homotopy perturbation method (HPM) is a mixture of the perturbation method and the homotopy method that has overcome the constraints of classic perturbation approaches. For additional situations, the series Equation A9 is convergent.

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Appendix B

```
% Parameters (to be defined as per the system)
v = 0.1;       % Growth rate of N
beta = 0.5;    % Transmission rate
mu = 0.02;     % Natural death rate
u1 = 0.01;     % Control on S
u2 = 0.01;     % Control on I
gamma = 0.1;   % Recovery rate
alpha = 0.05;  % Disease-induced death rate
% Initial Conditions
S0 = 0.9;  % Initial susceptible population
I0 = 0.16; % Initial infected population
R0 = 0;    % Initial recovered population
N0 = 1;    % Total initial population
% Time Span
tspan = [0 50]; % Simulate from t = 0 to t = 50
% Define the System of ODEs
odes = @(t, y) [
   v*y(4) - beta*y(1)*y(2) - (mu + u1)*y(1);                    % S'(t)
   beta*y(1)*y(2) - (gamma + mu + alpha)*y(2) - u2*y(2);        % I'(t)
   gamma*y(2) - mu*y(3) + u1*y(1) + u2*y(2);                    % R'(t)
   v*y(4) - alpha*y(2) - mu*y(4)                                % N'(t)
];
% Solve the System of ODEs
y0 = [S0; I0; R0; N0];
[t, y] = ode45(odes, tspan, y0);
% Extract Results
S = y(:, 1);
I = y(:, 2);
R = y(:, 3);
N = y(:, 4);
% Plot Results
figure;
% Plot S(t)
subplot(2, 2, 1);
plot(t, S, 'b', 'LineWidth', 1.5);
xlabel('Time t');
```

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```
ylabel('S(t)');
title('Susceptible Population');
grid on;
% Plot I(t)
subplot(2, 2, 2);
plot(t, I, 'r', 'LineWidth', 1.5);
xlabel('Time t');
ylabel('I(t)');
title('Infected Population');
grid on;
% Plot R(t)
subplot(2, 2, 3);
plot(t, R, 'g', 'LineWidth', 1.5);
xlabel('Time t');
ylabel('R(t)');
title('Recovered Population');
grid on;
% Plot N(t)
subplot(2, 2, 4);
plot(t, N, 'k', 'LineWidth', 1.5);
xlabel('Time t');
ylabel('N(t)');
title('Total Population');
grid on;
```

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