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Semi - Analytical Solution of Modelled Typhoid Fever Disease

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I N F O

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A B S T R A C T

To analyze the optimal control of the Typhoid fever virus a mathematical modeling was developed by Getachew Teshome Tilahun. We have approached a Homotopy Perturbation Method to solve a linear differential equation. An analytical solution of Susceptible People(S), Infected People (I), Carrier People (C), Recovered People (R), and Bacteria People (is obtained and compared with simulation results. A significant agreement is produced when approximate analytical results are compared to numerical simulation. The treatment rate of infectious disease (, Natural death rate (and typhoid-induced death rates (α) are discussed.

Keywords: Typhoid Modeling, Bacteria Populations, Human Population, Analytical Solution, Homotopy Perturbation Method, and Numerical Simulation

Introduction

The signs and symptoms of typhoid fever are similar to those of typhus, which was the given that name. It's a human-borne endemic infectious disease caused by a highly aggressive and "invasive Salmonella Enterica Serovar Typhi (S. Typhi)" strain. Supplies of food tainted with an infected patients or a carrier's feces and urine can spread this pathogen.[1] Persistent fever, nausea, vomiting, a terrible headache, and weariness are some of the signs and symptoms. Typhoid fever has a 7-14-day incubation period. For certain enteric bacteria, the intestine is a natural environment. The bacteria grow in "Mononuclear Phagocytic cells" during acute infection before being discharged into the bloodstream.[2]

Antibiotics can be used to treat typhoid fever. According to[3] the disease can be avoided by getting vaccinated with "An injectable typhoid conjugate vaccine (TCV); an injectable polysaccharide vaccine based on purified Vi antigen for children aged 2 years and up; and an oral live attenuated Ty2Ia vaccine in capsules for children aged 6 years and up. Among the three vaccinations mentioned above, (TCV) is the most popular since it may be given to people of all ages, from infants to adults, and it has better immunological properties". Typhoid is a serious public health issue in Kenya[4] for children under the age of 15. Because they are prone to playing with rainfall, they have a greater chance of contracting the disease. According to the global burden of diseases, "Kenya reported 97,767 typhoid cases in 2016, with 62 percent of those being children under the age of 15, and 1075 typhoid cases deaths, with 66 percent being children under the age of 15".

To represent the dynamics of the spread of typhoid, several mathematical models have been developed. Nthiiri et al.[5] discovered"a model to analyze the dynamics of typhoid fever using protection as a management approach, and they discovered that increasing protection would significantly lower disease prevalence in a community based on numerical simulation findings". Stephen and Nyerere[6] devised a model for assessing "the effectiveness of educational initiatives in preventing typhoid transmission in the community. Based on their examination of the model, they discovered that raising awareness might reduce transmission by more than 40%"."Many researchers have created mathematical models to explore the dynamics of typhoid fever while considering a variety of control techniques, including vaccine, therapy, and screening".[7]

Until 2017, there is no study has been done to investigate typhoid fever dynamics with the application of optimal control methods and cost-effectiveness analysis of the applied control strategies. So, we have worked on this data initially to solve it analytically.

He and El Dib[8] used the homotopy perturbation method to solve "the solitary Duffing-like oscillator issue more effectively". Naveed and He[9] used "The homotopy perturbation and He-Laplace technique to solve nonlinear issues in nano/microelectromechanical systems". By using the parameter expansion method, Yue Wu and He[10] demonstrated that "the homotopy perturbation method could be effectively applied to nonlinear oscillators with no linear term or a negative linear term".

The Salmonella Typhi bacteria causes typhoid fever. Typhoid fever is the most common bacterial disease in Asian countries, and India is one of the major countries. It easily spreads with poor sanitation. These variables (S, I, C, R and) represent the number of people in each compartment at a particular time. To represent that the number of susceptible, infectious, Carrier Humans, removed individuals, and Bacteria populations may vary over time (even if the total population size remains constant), we make the precise numbers a function of t (time): S(t), I(t), C(t), R(t) and (t). For a specific disease in a specific population, these functions may be worked out to predict possible outbreaks and bring them under control.

Mathematical Formulation of Typhoid Fever

Getachew Teshome Tilahun, Oluwole Daniel Makinde, and David Malonza[11] were developed the "Modelling and Optimal Control of Typhoid Fever Disease with Cost-Effective Strategies". The formulation of the model is classified into human population and bacteria population. The population of humans and bacteria is divided into four subclasses Susceptible Humans (S), Infected Humans (I), Carrier Humans (C), Recovered Humans (R), and Bacteria Population (B_c) The formulation is described as follows:

$$\frac{dS}{dt} = \Lambda + \delta R - (\mu + \lambda)S (1)$$

$$\frac{dC}{dt} = \rho\lambda S - (\sigma_1 + \theta + \mu + \phi)C (2)$$

$$\frac{dI}{dt} = (1-\rho)\lambda S + \theta C - (\sigma_2 + \beta + \mu + \alpha)I (3)$$

$$\frac{dR}{dt} = \beta I + \phi C - (\mu + \delta)R (4)$$

$$\frac{dB_c}{dt} = \sigma_1 C + \sigma_2 I - \mu_b B_c (5)$$

The initial conditions are:

$$S(0) = S_0 \geq 0, C(0) = C_0 \geq 0, I(0) = I_0 \geq 0, R(0) = R_0 \geq 0, B_c = B_{c0} \geq 0 (6)$$

$$\lambda = \frac{B_c \nu}{(K + B_c)}.$$

Analytical Solution of Typhoid Fever

In this section, a new approach, homotopy perturbation method is used to solve analytical solution of equation (1) - (4), the expand derivative of HPM is discussed in appendix A.

$$\frac{dS}{dt} - \Lambda - \delta R + (\mu + \lambda)S = 0 (7)$$

$$\frac{dC}{dt} - \rho\lambda S + (\sigma_1 + \theta + \mu + \phi)C = 0 (8)$$

$$\frac{dI}{dt} - (1-\rho)\lambda S - \theta C + (\sigma_2 + \beta + \mu + \alpha)I = 0 (9)$$

$$\frac{dR}{dt} - \beta I - \phi C + (\mu + \delta)R = 0 (10)$$

$$\frac{dB_c}{dt} - \sigma_1 C - \sigma_2 I + \mu_b B_c (11)$$

Getachew Teshome Tilahun et al. developed this model in 2017. In this, we have used Maple software for an analytical solution and MATLAB software for numerical solutions.

Now,

$$(1-V)\left(\frac{dS}{dt} - \Lambda + (\mu + \lambda)S\right) + V\left(\frac{dS}{dt} - \Lambda - \delta R + (\mu + \lambda)S\right) = 0 (12)$$

$$(1-V)\left(\frac{dC}{dt} + (\sigma_1 + \theta + \mu + \phi)C\right) + V\left(\frac{dC}{dt} - \rho\lambda S + (\sigma_1 + \theta + \mu + \phi)C\right) = 0 (13)$$

$$(1-V)\left(\frac{dI}{dt} + (\sigma_2 + \beta + \mu + \alpha)I\right) + V\left(\frac{dI}{dt} - (1-\rho)\lambda S - \theta C + (\sigma_2 + \beta + \mu + \alpha)I\right) = 0 (14)$$

$$(1-V)\left(\frac{dR}{dt} + (\mu + \delta)R\right) + V\left(\frac{dR}{dt} - \beta I - \phi C + (\mu + \delta)R\right) = 0 (15)$$

$$(1-V)\left(\frac{dB_c}{dt} + \mu_b B_c\right) + V\left(\frac{dB_c}{dt} - \sigma_1 C - \sigma_2 I + \mu_b B_c\right) = 0 (16)$$

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The solution of equation (12) - (16) is expressed in power series:

$$S = S_0 + VS_1 + V^2S_2 + V^3S_3 + \cdots (17)$$
$$C = C_0 + VC_1 + V^2C_2 + V^3C_3 + \cdots (18)$$
$$I = I_0 + VI_1 + V^2I_2 + V^3I_3 + \cdots (19)$$
$$R = R_0 + VR_1 + V^2R_2 + V^3R_3 + \cdots (20)$$
$$B_c = B_{c0} + VB_{c1} + V^2B_{c2} + V^3B_{c3} (21)$$

Equation (17)-(13) into Equation (12)-(16) and arranging the coefficients of the powers of V produce the following systems of differential equations:

$$V^0: \frac{dS_0}{dt} - \Lambda + (\mu + \lambda)S_0$$
$$V^1: \frac{dS_1}{dt} + (\mu + \lambda)S_1 - \delta R_0 (22)$$
$$V^2: \frac{dS_2}{dt} + (\mu + \lambda)S_1 - \delta R_1$$
$$V^0: \frac{dC_0}{dt} + (\sigma_1 + \theta + \mu + \phi)C_0$$
$$V^1: \frac{dC_1}{dt} + (\sigma_1 + \theta + \mu + \phi)C_1 - \rho\lambda S_0 (23)$$
$$V^2: \frac{dC_2}{dt} + (\sigma_1 + \theta + \mu + \phi)C_2 - \rho\lambda S_1$$
$$V^0: \frac{dI_0}{dt} + (\sigma_2 + \beta + \mu + \alpha)I_0$$
$$V^1: \frac{dI_1}{dt} - (1 - \rho)\lambda S_0 - \theta C_0 + (\sigma_2 + \beta + \mu + \alpha)I_1 (25)$$
$$V^2: \frac{dI_2}{dt} - (1 - \rho)\lambda S_1 - \theta C_1 + (\sigma_2 + \beta + \mu + \alpha)I_2$$
$$V^1: \frac{dR_1}{dt} + (\mu + \delta)R_1 - \beta I_0 - \phi C_0 (24)$$
$$V^2: \frac{dR_2}{dt} + (\mu + \delta)R_2 - \beta I_1 - \phi C_1$$
$$V^0: \frac{dB_{c0}}{dt} + \mu_b B_{c0}$$
$$V^1: \frac{dB_{c1}}{dt} + \mu_b B_{c1} - \sigma_1 C_0 - \sigma_2 I_0 (26)$$
$$V^2: \frac{dB_{c2}}{dt} + \mu_b B_{c2} - \sigma_1 C_1 - \sigma_2 I_1$$

Analytical Expression of Typhoid Fever Model

$$S(t) = 0.0809 + 0.91908e^{(-0.02471790000t)} (27)$$

$$C(t) = 0.9999967522e^{-1.125000000*t} + 0.00000025748 + 0.00000299e^{-0.02471790000t} (28)$$

$$R(t) = 1.002617279e^{(-.25604t)} - 0.00234440204e^{(-0.8787t)} - 0.0002728771071e^{(-1.125t)} (30)$$

$$B_c(t) = 2.874368750e^{(-0.001t)} - 0.2227604829e^{(-1.125t)} - 1.651592306e^{-0.8787t} + 0.00057739 - 0.0005933469620e^{-0.024718t} \quad (31)$$

Numerical Results and Discussion

Equations (27) - (31) represents the analytical expressions of the typhoid fever model. The numerical solution and the resultant analytical results are contrasted, which is represented graphically in Figure (1) - (5).

Table 1.Parameter Values and Description of Typhoid Fever Model

Symbol	*Description*	*Values*	*Source*
v	Salmonella Ingestion Rate	0.9	[11]
K	The concentration of Salmonella Bacteria in Foods and Water	50000	[11]
μ	Human Beings Natural Death Rate	0.0247	[11]
α	Typhoid Induced Death rate	0.052	[11]
β	TreatmenttRate of Infectious Disease	0.002	[11]
σ_1	Discharge Rate of Salmonella from Carriers	0.9	[11]
σ_2	Discharge Rate of Salmonella from Inactive	0.8	[11]
δ	Removal Rate from Recovered Subclass to Susceptible Subclass	0.000904	[11]
θ	Screening Rate oftCarriers	0.2	[11]
ϕ	Removal of Carriers by Natural Immunity	0.0003	[11]
ρ	Probability of Susceptible Joining Carrier State	0.3	[11]
μ_b	Natural/Drug-Induced Death Rate oftBacteria	0.001	[11]
Λ	Recruitmenttof Human Beings	100	[11]

In Figure 1 and 2, the infected populations transmission rate or treatment rate is decreased due to changing the parameter values in higher range into β=0.002,0.09,0.9,2 and 5. So, the transmission rate is reduced if we use the proper prevention and follow hygienic life. In infected population, the typhoid induced death rate is decreased if we increase the values of α into 0.05, 0.9, 5 and 15. With safe drinking water, sanitary facilities, and sufficient medical care, typhoid fever can be managed and controlled. Unfortunately, in many developing countries, accomplishing these objectives may be difficult. As a result, some researchers believe that immunizations are the most efficient way to prevent

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typhoid.

In figure 3, the recovered population's rate at which a recovered subclass is transferred to a vulnerable subclass is decreasing while we change the parameter values into δ =0.000904, 0.005, 0.01 and 0.05. Then, the bacteria populations of natural/drug induced death rate of bacteria

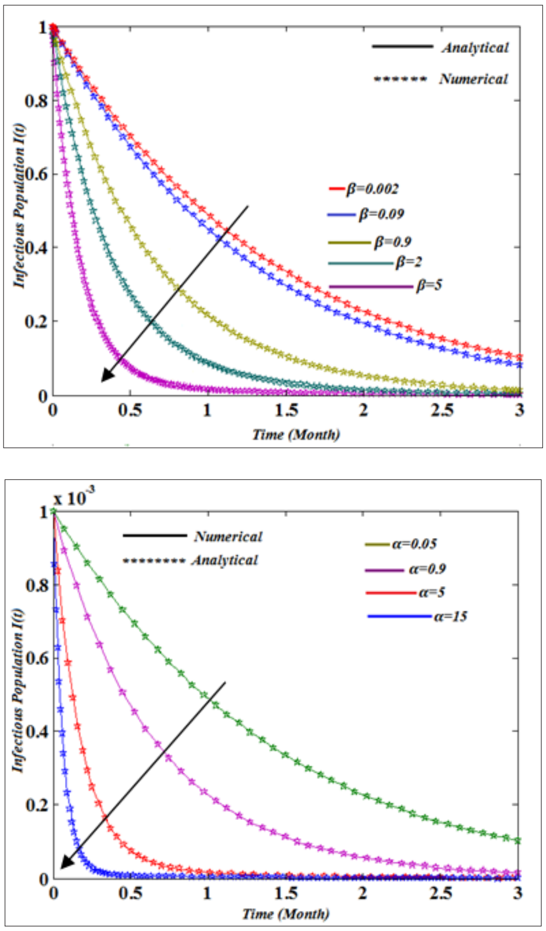

Figure 1 and 2.Variation of infected populations transmission rate and induced death rate

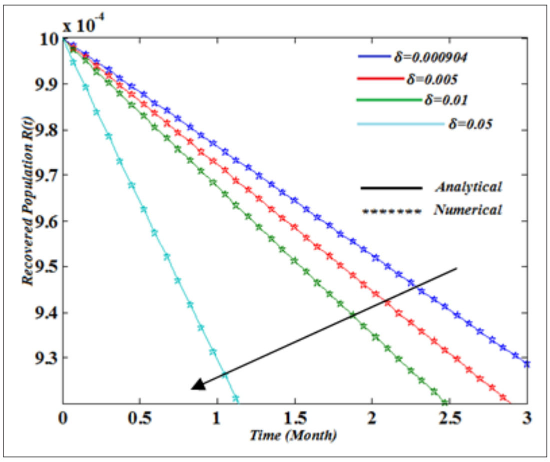

Figure 3.Variations of removal rate of δ

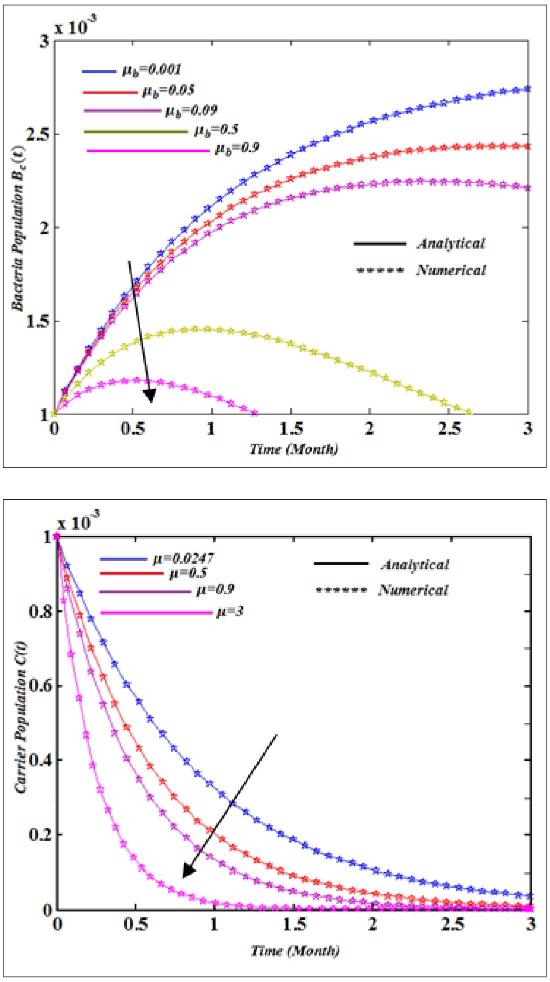

Figure 4 and 5.Human Beings natural death rate and bacteria death rate

μ_b are as shown in figure 4. The values of μ_b is increased by changing the values into 0.001, 0.05, 0.09, 0.5 and 0.9. In figure 5, the carrier population of human beings' natural death rate is decreased when we increase the values of parameter μ into 0.0247, 0.5, 0.9 and 3. So, by increasing the values transmission rate, natural death rate, bacterial death rate into a slightly higher range, the rate usually goes down. So, if we follow proper preventions and healthy habits it means that we can overtake this virus, for example, we should be aware about the problems of open water, open foods and use fiber nets etc. as preventions.

Conclusion

The study on investigating typhoid fever dynamics with the application of optimal control methods and a cost-effectiveness analysis of the applied control strategies was not analyzed till 2017. This model is perfectly suited for optimal control of various bacterial diseasesover the other model. The nonlinear differential equation is solved using the analytic approach of homotopy perturbation method. In this paper, the typhoid fever model is solved analytically by using HPM method and the results accord well with the

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numerical and experimental findings. The MATLAB ode 45 function generates the numerical results that are consistent with the analytical solution by Homotopy Perturbation Method. We have represented parameters values flowing graphically in MATLAB. The variation of parameters in thegraph is concluding the possibilities of transmission rate, natural death rate and bacteria death rate goes high or low, while the range of parameter values increase.

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Conflict of Interest: None

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Appendix (I)

The Homotopy Perturbation Method's Fundamental Principle

Consider the following non-linear functional equation to demonstrate the essential elements of this technique:

$$X(u) - Y(v) = 0, s \in \Omega(\mathrm{A1})\mathrm{U}\left(u, \frac{\partial u}{\partial t}\right) = 0, v \in \Gamma \quad (\mathrm{A2})$$

X indicated functional operator, Y is a boundary operator, Y(v) for a known analytical equation, and Ω the limit of the domain Γ . M and R, with M denoting linear and R denoting nonlinear.

$$M(u) + R(u) - Y(v) = 0(\mathrm{A3})$$

We create a homotopy equations using the homotopy procedure,

$$K(u, V) = (1 - V)[M(u) - M(u_0)] + V(X(u) - Y(v)] = 0, V \in [0,1], v \in \Omega(\mathrm{A4})$$

V $\in$ [0,1] is a parameter, for the solution of Equation, u_0 is an initial approximation. (A2), which meets the border requirements. Clearly, we may deduce the following from Eqns. (A4):

$$K(u, 0) = M(u) - M(u_0) = 0 \ (\mathrm{A5})$$
$$K(u, 1) = X(u) - Y(v) = 0 \ (\mathrm{A6})$$

Altering V's value from zero to unity corresponds to changing *u(v,V)* value from u_0 *(v)* to *u(v)*. This is known as homotopy in topology. The embedding parameter can be used as a tiny parameter, and the solution of Equations (A4) and (A5) can be assumed to be a power series in V:

$$U = u_0 + Vu_1 + V^2u_2 + .. \ (\mathrm{A7})$$

Setting *V* = 1, approximation to Eqn's solution. (A7)

$$U = u_0 + Vu_1 + V^2u_2 + ... \quad (\mathrm{A7})$$

The homotopytperturbationtmethod (HPM) is a mixture of thetperturbation methodtand thethomotopy method that has overcome the constraints of classic perturbation approaches. For additional situations, the series Eqn. (A8) is convergent.

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